Continuous Global N-Tuple Coverage with $(2N+2)$ Satellites

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N-tuple continuous, global coverage of a spherical planet is shown to be possible, using $(2n+2)$ inclined elliptic orbit satellites. A simple proof is presented, showing that n-tuple coverage is impossible, either instantaneously or continuously, with fewer than $(2n+2)$ satellites. Examples of single and redundant continuous global coverage constellations developed by the author incorporate rotating, irregular (warping) tetrahedra or prismoids. The constellations are characterized by two groups of satellites, each having approximately circular, overlapping ground tracks on a pseudoplanet having the same period as the satellites. One group of satellites has perigees in the southern hemisphere and their ground tracks on the pseudoplanet appear to rotate counterclockwise. The other group has its perigees in the northern hemisphere and appears to rotate clockwise. A generalized table of orbital parameters for n-tuple coverage constellations is presented.

Nomenclature

A	= semimajor axis of satellite orbit, in units of the Earth's equatorial radius r
a	= semimajor axis, n.mi.
e	= eccentricity
h	= satellite altitude, n.mi.
i	= inclination, deg
M	= mean anomaly, deg
P	= semiparameter, $=a(1-e^2)$
p	= perpendicular distance from Earth center to satellite plane, n.mi.
R_e	= mean radius of Earth, $=3442$ n.mi
r	= satellite radius, n.mi.
S_1,S_2,S_3,S_4	= satellite designator
S_1',S_2',S_3',S_4'	= satellite suborbital points
T	= constellation period, h
μ	= Earth's gravitational constant, $=1.4077\times10^{16}$ ft^3/s^2
σ	= satellite visibility angle ("look angle") above horizon, deg
Ω	= right ascension of ascending node, deg
ω	= argument of perigee, deg

Background

THE minimum number of satellites required for continuous global single, double, triple, and quadruple coverage, respectively, using circular orbit Walker-type constellations, are 5, 7, 9, and 11.[1] The first two four-satellite continuous global single-coverage arrays were developed by the author in 1984 and 1985.[2-4] The first six-satellite continuous global double-coverage array was developed in 1986. Continuous triple- and quadruple-coverage arrays using 8 and 10 satellites, respectively, were developed in 1988.

Introduction

The level of dependence on multisatellite ordered constellations, or arrays, for a variety of military and civil requirements is increasing every year. Some of these requirements, or missions, include surveillance, communications, data relay, navigation, meteorology, and air traffic control. It is not uncommon to find individual satellites in such a system costing in excess of 100 million dollars, with the booster necessary to place it in orbit having a like cost. Obviously, any means possible for minimizing the number of satellites (and boosters) to provide single or redundant continuous Earth coverage could bring about major cost savings. The use of elliptic orbit constellations can provide more efficient coverage with fewer satellites and without gaps, in many instances, than is possible using circular-orbit, Walker-type constellations.

Several theorems and corollaries have previously been presented that are of assistance in developing and understanding the satellite coverage principles involved in elliptic orbit constellations.[2,3] In dealing with continuous redundant-coverage systems, rather than continuous single-coverage systems, it was found necessary to develop additional theorems and corollaries. These theorems and corollaries are grounded in solid geometry, as were the previous ones. Although they are simple to understand and apply, they are only the first step towards the full development and optimization of these optimal-satellite constellations. Optimization would have been impossible to achieve without the availability of the modern, high-speed microcomputer. The majority of the work performed in the development of the triple and quadruple continuous coverage arrays was carried out at Science and Technology Associates, Inc. (STA) using an Apple Macintosh II PC, in conjunction with an Apple Laserwriter II printer.

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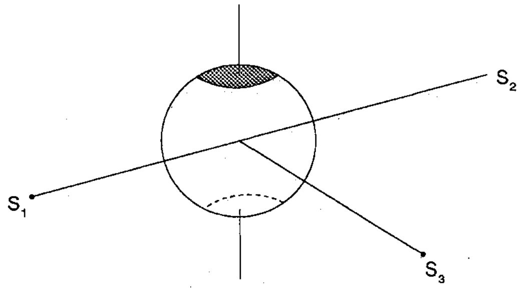

Fig. 1 Theorem III.

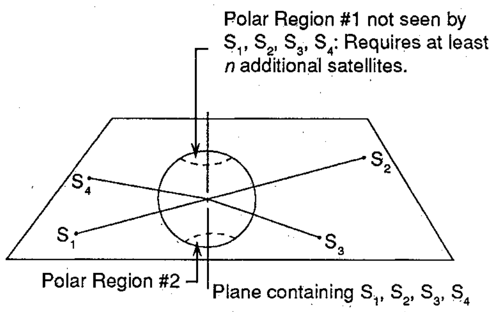

Fig. 2 Corollary III.

Since continuous coverage implies complete independence from an observer's geographic location, it has been found useful to employ the concept of a "pseudoplanet" whose period is the same as the constellation's period. This has the effect of yielding repeating ground tracks (i.e., the traces of the subsatellite points on the surface of the pseudoplanet). The familiar figure-eight, circular, elliptic, or tear-drop patterns appear for inclined orbits. Linear or point traces appear for orbits with zero inclination, for elliptic or circular orbits, respectively. The visualization of the coverage patterns thus become easier for the constellation designer, since coverage patterns are repeated for each constellation period, and critical or fringing coverage patterns are localized in predictable zones or areas.

Throughout this study, a spherical Earth or planet is assumed.

Theorems and Corollaries

The following theorems and corollaries supplement those presented in Refs. 2 and 3. They are useful in addressing the problem of redundant continuous coverage. (Theorem I and its corollaries appear in Refs. 2 and 3 and apply to global continuous coverage requirements. Theorem II and its corollaries apply only to hemispheric continuous coverage and appear in Ref. 2.)

Theorem III. If two satellites in a multisatellite constellation are diametrically opposite from one another, a minimum of $(2n+3)$ satellites is required to obtain instantaneous global n-tuple coverage. (Satellites are assumed to be at a finite altitude.)

Proof. A great circle may be passed through the two satellites, which are diametrically opposite, and any third satellite. At least $2n$ additional satellites are then needed in order to obtain n-tuple coverage of the two poles normal to the great circle plane. (See Fig. 1).

Corollary III. If m satellites are coplanar with a planet's center, then a minimum of $(m+2n)$ satellites is required to obtain instantaneous n-tuple coverage.

Proof. A great circle may be passed through the m satellites. The two poles to the great circle require an additional $2n$ satellites, at a minimum, to obtain n-tuple coverage. (See Fig. 2).

Theorem IV. For a system of synchronous elliptic orbit satellites to have identical ground tracks, the satellites must

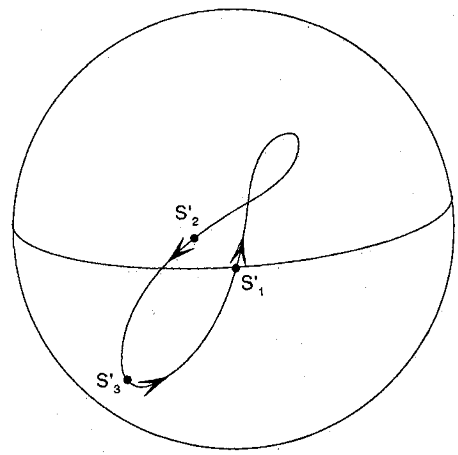

Fig. 3 Theorem IV.

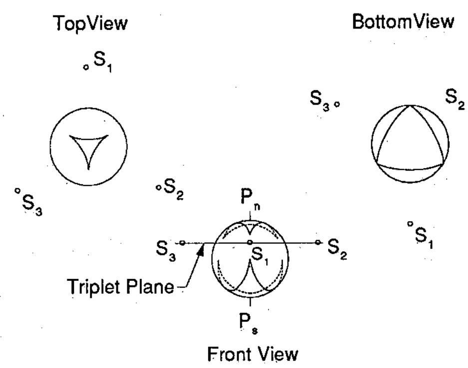

Fig. 4 Theorem V.

have the same inclination, eccentricity, period, and argument of perigee, and the sum of the right ascension of the ascending node and the satellite mean anomaly must be a constant for each satellite in the system for any time instant.

Proof. Since all of the satellites' parameters except right ascension of the ascending nodes and mean anomalies are identical, the shape of all of the orbital ground tracks are identical, save for a possible longitudinal displacement. Equatorial crossings will occur at the same longitude, resulting in overlying ground tracks, provided any angular increase in the right ascension of a given satellite's line of nodes is offset by an equal but opposite angular decrease of the mean anomaly for that same satellite. Subtracting the mean anomaly for a synchronous system effectively subtracts out the planetary rotation rate and converts the inertial reference frame to a planetary reference frame. Constraining the sum of the right ascension of the ascending node plus the mean anomaly to a unique constant value ensures that any increase in right ascension of the ascending node will occasion a corresponding decrease in the mean anomaly. Then, at earlier or later points in time where the other satellites in the system cross the equator at their respective ascending nodes, their mean anomalies will be the same as that of the original reference satellite. The crossing longitudes will thus be the same because the shape of every satellite's ground track is identical (with corresponding mean anomalies matching with the latitude crossings). (See Fig. 3.)

Theorem V. If three satellites form a plane that intersects a spherical planet, then a minimum of $(2n+3)$ satellites is required to obtain instantaneous global n-tuple coverage.

Proof. Erect a perpendicular to the triplet plane through the planet's center. We will define the poles as the intersections of this perpendicular diameter with the surface of the planet. It may easily be seen that at each pole there is no visibility by any of the triplet satellites (assuming these satellites are at finite altitudes). Thus, n additional satellites are needed to

Table 1 Starting positions, single-coverage constellation; $n = 1$

Satellite number	T, h	i, deg	e	ω, deg	Ω, deg	M, deg
1	= >26.49	31.3	0.263	−90	0	0
2	= >26.49	|	|	+90	−90	+90
3	= >26.49	|	|	−90	−180	+180
4	= >26.49	↓	↓	+90	−270	+270

provide instantaneous n-tuple coverage of each of these poles. The minimum total number of satellites required is, therefore, $(2n + 3)$, representing the three satellites in the intersecting plane and $2n$ more for the two poles. (See Fig. 4.)

Common Period Continuous Single Coverage Array[5]

The minimum-satellite array providing continuous single coverage consists of four satellites arranged in a tetrahedral geometry (Fig. 5). The orbital parameters for each satellite in the array are listed in Table 1. Satellites in the array are organized into two pairs where both satellites in a pair follow an identical ground track. The two pairs begin an orbital period on opposite hemispheres of the earth and are skewed in relation to each other; a line joining together the first pair would run north-south, whereas a line joining the second pair's satellites would run east-west.

Figure 6 illustrates a plot of the loci of satellite positions over an orbital period as seen by an observer rotating in the pseudoplanetary reference frame (the pseudoplanet shown has radius of Earth and is drawn to scale). The frontal view shows clearly the front hemisphere satellite pair loci (perigees in southern hemisphere) and the back hemisphere satellite pair loci (perigees in northern hemisphere). Each pair appears as a single oval shape. It is interesting to note that the side view shows a nearly linear path followed by each satellite pair.

Proof of global coverage was accomplished by rigorous examination of instantaneous visibility limit plots for the four-satellite array at discrete intervals spanning an orbital period. Figure 7 shows a rotated visibility limit plot for the four-satellite array at the start of its common orbital period. The satellite groups have been rotated 90 deg, so that they are centered over the north and south poles. The reason for doing this is to reduce distortion of the fringing area patterns when plotted on a rectangular or a Mercator chart. In actuality, the fringing patterns will lie close to two opposing meridians of longitude. The best coverage areas will lie within the two circular ground tracks, where the degree of coverage will be at least $(n + 1)$. Figure 8 follows the changing limit curves over an orbital period with a plot taken at every 30-deg interval of mean anomaly. Lowering the array to a synchronous altitude would result in a slightly degraded coverage pattern in which small gaps of noncoverage begin to appear, but most of the globe would still be continuously visible to the array. The minimum period sufficient for global single coverage is 26.49 h. This value was determined by a computer program that first calculates the perpendicular distance from the center of a planet to the plane of a representative satellite triplet. The program uses a grid-search technique, for various combinations of satellite eccentricity and inclination, to find the combination that yields the minimum possible period that will still avoid intersection of the plane with the surface of the spherical planet.

The structure of the array insures that, with sufficient altitude, no plane constructed from any three of the four

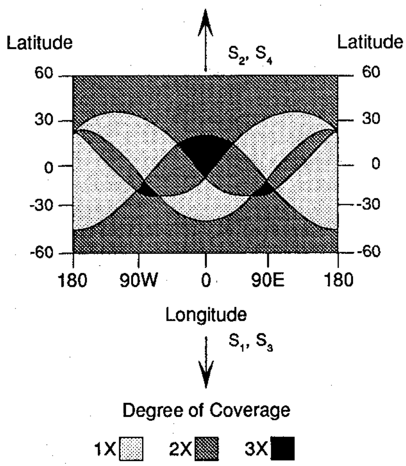

Fig. 7 Four-satellite visibility limits, starting position of constellation.

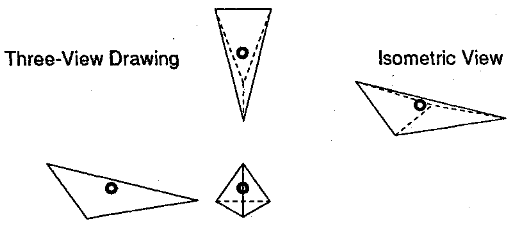

Fig. 5 Four-satellite tetrahedral constellation for continuous single coverage.

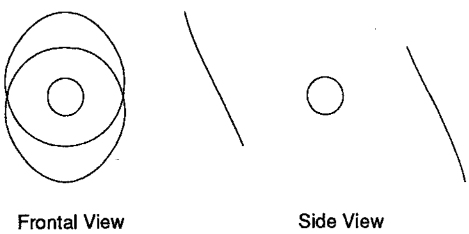

Fig. 6 Four-satellite tetrahedral array (frontal and side views in pseudoplanet reference frame).

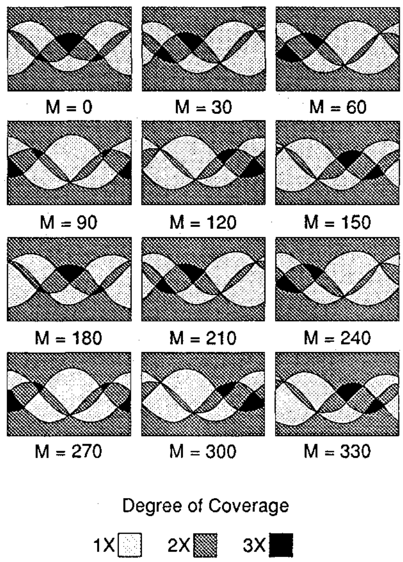

Fig. 8 Four-satellite visibility limit plots at 30-deg intervals of mean anomaly.

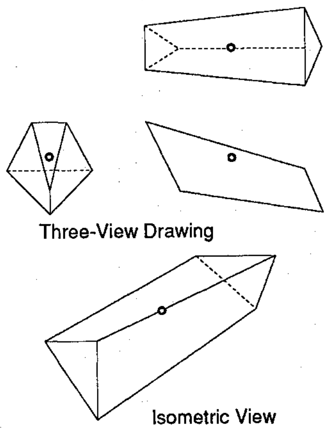

Fig. 9 Six-satellite triangular prismoid for continuous double coverage.

satellites in the array will intersect the Earth's surface. Such a tetrahedral geometry is unique to the single-coverage model (at higher levels of coverage, minimum-satellite constellation geometries are prismoidal), although the structures of minimum-satellite arrays with higher levels of coverage have the same property of nonintersection.

Common Period Continuous Double-Coverage Array

The minimum-satellite array providing continuous double coverage[4] consists of six satellites arranged in a triangular prismoidal geometry (Fig. 9). This array is the simplest prismoidal array of the family being described. The orbital parameters for each satellite in the array are listed in Table 2. Satellites are organized into two sets of three satellites each, with each set composing one end of the triangular prismoid. The ends of the prismoid are congruent, unlike the four-satellite model, which has skewed pairs. Each set of satellites shares a ground track, so that two ground tracks represent the entire array of satellites.

The side and frontal views of the double-coverage array are shown in Fig. 10. Proof of coverage was accomplished in the same manner as the single-coverage case, by plotting the satellite visibility limits over an orbital period (Fig. 11). The minimum period providing a sufficient constellation altitude for global double coverage is 102 h.

Observations on Single- and Double-Coverage Arrays

The independent discoveries of the four-satellite common period tetrahedral array for continuous global single coverage and the six-satellite prismoidal array for continuous double coverage had only one obvious common link. This was the existence of two counter-rotating sets of roughly circular ground tracks, one set on either side of the pseudoplanet. The phasing, however, was quite different. The four-satellite array assumed a cruciform shape when viewed axially. That is, the two satellites on the far side "split the difference" in the spacing on the near side. By contrast, in the six-satellite double-coverage array, the spacings were in parallel, or in alignment. That is, the triangles, representing the two bases of the prismoid, were lined up base to base and vertex to vertex at their starting positions. This fact led introspectively, to the formulation of Theorem III, when it was observed that splitting the difference in spacing with two triangular bases would never achieve the desired double coverage. At the starting position, one key satellite would be at apogee and one satellite (on the other side) at perigee. They would, at that instant, be on opposite ends of a spherical diameter, making the desired double coverage with six satellites impossible, in accordance with Theorem III. (This "skewed" prismoid with opposed triangular bases and 90 deg satellite spacings is actually an octahedron.) Yet, a regular triangular prism with parallel sides does have the capability of providing instantaneous double coverage, as described in Ref. 4.

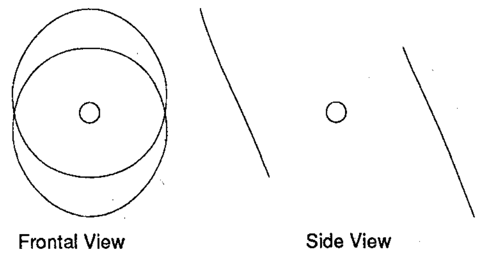

Fig. 10 Six-satellite triangular prismoidal array (frontal and side views in pseudoplanet reference frame).

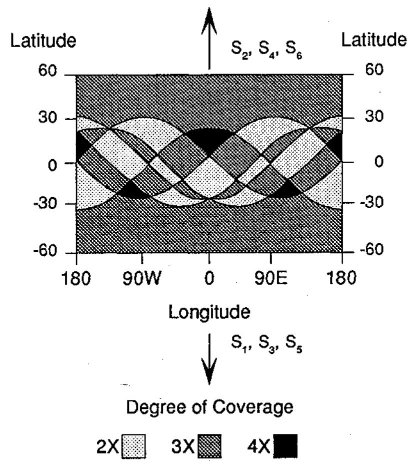

Fig. 11 Six-satellite visibility limits, starting position of constellation.

Development of *N*-Tuple Coverage Arrays

To avoid the condition of finding satellites lying at opposite ends of a diameter, the author developed the strategy of employing either "skew" or "congruent" prismoidal arrays, depending on whether n were even or odd, respectively. The increase of n to three (triple coverage) or four (quadruple coverage) would then appear tractable using a skew prismoid or a congruent prismoid, respectively. This was, in fact, found to be a valid observation, and there is little reason to doubt its generality for all values of n.

Table 2 Starting positions, double-coverage constellation; $n = 2$

Satellite number	T, h	i, deg	e	ω, deg	Ω, deg	M, deg
1	=> 102	27.5	0.233	−90	0	0
2	=> 102	|	|	+90	−60	+60
3	=> 102	|	|	−90	−120	+120
4	=> 102	|	|	+90	−180	+180
5	=> 102	|	|	−90	−240	+240
6	=> 102	↓	↓	+90	−300	+300

Table 3 Starting positions, triple-coverage constellation; $n = 3$

Satellite number	T, h	i, deg	e	ω, deg	Ω, deg	M,deg
1	= >272	25	0.218	−90	0	0
2	= >272	↓	↓	+90	−45	+45
3	= >272			−90	−90	+90
4	= >272			+90	−135	+135
5	= >272			−90	−180	+180
6	= >272			+90	−225	+225
7	= >272			−90	−270	+270
8	= >272	↓	↓	+90	−315	+315

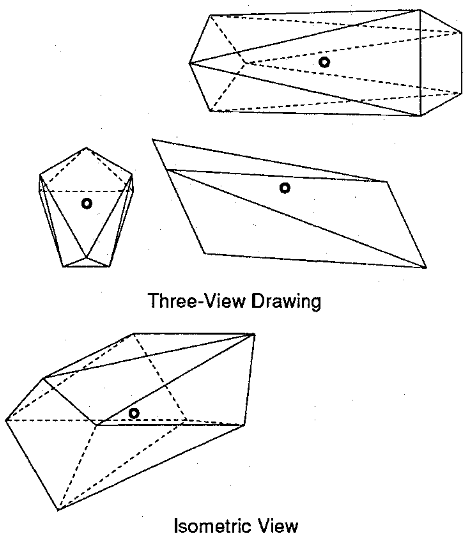

Fig. 12 Eight-satellite quadrangular prismoid for continuous triple coverage.

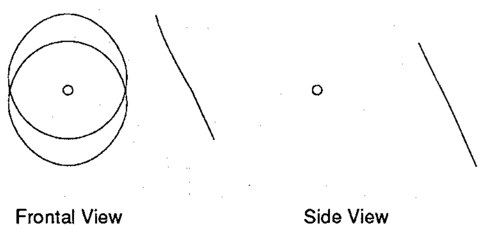

Fig. 13 Eight-satellite quadrangular prismoidal array (frontal and side views in pseudoplanet reference frame).

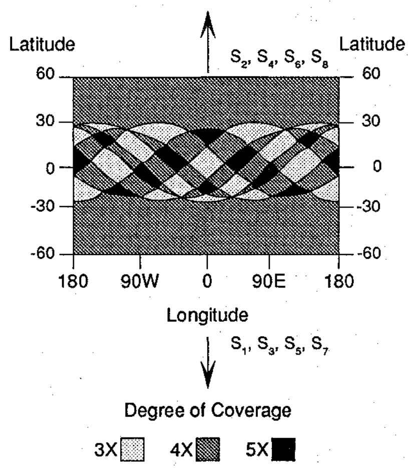

Fig. 14 Eight-satellite visibility limits, starting position of constellation.

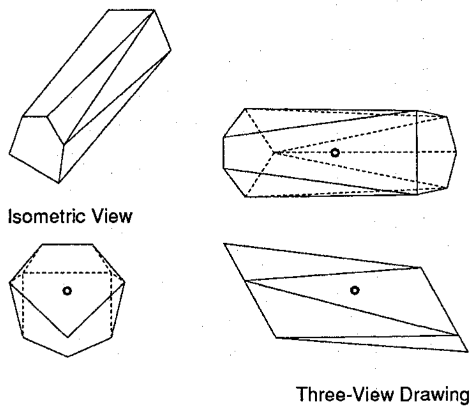

Fig. 15 10-satellite pentagonal prismoid for continuous quadruple coverage.

Common Period Continuous Triple Coverage Array

Global triple coverage is achieved by arranging eight satellites in a quadrangular prismoidal geometry (Fig. 12). The orbital parameters are outlined in Table 3. These satellites are arranged into two sets of four, with each set sharing a single ground track. Each set of four satellites represents one end of the quadrangular prismoid. The two ends are skewed, as in the four-satellite model. This constellation requires a minimum period of 272 h.

The eight-satellite array also produces only two ground tracks. The frontal and side views for the eight-satellite array (Fig. 13) show how far out the satellites must be to provide triple coverage (the pseudoplanet is still drawn to scale).

The visibility limit plot for the eight-satellite array's starting position is illustrated in Fig. 14.

Common Period Continuous Quadruple-Coverage Array

Global quadruple coverage is achieved by arranging 10 satellites in a pentagonal prismoidal geometry (Fig. 15). The orbital parameters are outlined in Table 4. These satellites are arranged into two sets of five, with each set sharing a single ground track. Each set of five constitutes one end of the pentagonal prismoid; the ends of this model are congruent.

The frontal and side views for the 10-satellite array are shown in Fig. 16. To provide quadruple coverage, this constellation requires a minimum period of 568 h, or almost 24 days. As is evident from the visibility limit plot in Fig. 17, this array is the most complex of the four that have been described.

Table 4 Starting positions, quadruple-coverage constellation; $n = 4$

Satellite number	T, h	i, deg	e	ω, deg	Ω, deg	M, deg
1	= >568	24	0.205	−90	0	0
2	= >568	│	│	+90	−36	+36
3	= >568	│	│	−90	−72	+72
4	= >568	│	│	+90	−108	+108
5	= >568	│	│	−90	−144	+144
6	= >568	│	│	+90	−180	+180
7	= >568	│	│	−90	−216	+216
8	= >568	│	│	+90	−252	+252
9	= >568	│	│	−90	−288	+288
10	= >568	↓	↓	+90	−324	+324

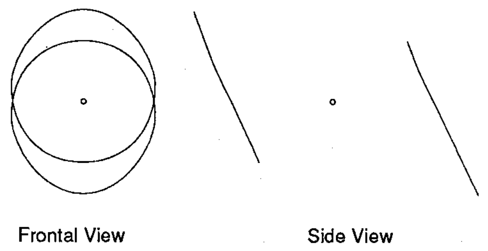

Fig. 16 10-satellite pentagonal prismoidal array (frontal and side views in pseudoplanet reference frame).

Satellite Periods, Eccentricities, Apogees, and Perigees

The quadruple-coverage 10-satellite array expresses the limit of our detailed investigation into the geometrical family of minimum-satellite arrays described in this paper. The minimum periods required to meet the redundant coverage levels above $n = 2$ mandate very high orbital altitudes where lunar and solar perturbations would threaten constellation integrity. Table 5 illustrates the great orbital radii involved for higher levels of coverage when compared to the single- and double-coverage models.

Generalized Table of Orbital Parameters

Development of a family of n-tuple coverage constellations led to the definition of a generalized table of orbital parameters (Table 6) that provides the proper phasing relationships. The table accounts for the skew/congruent relationship, meeting the requirements of Theorem IV. The indexed, generalized table of orbital parameters describes coverage arrays for all values of n. The only parameter values not derivable explicitly are the array's inclination i and eccentricity e. As n increases, the optimum values for i and e decrease. Using computer models, the actual optimal values of i and e (which yield the minimum possible period for the constellation) may be determined.

Benefits Derived from Tetrahedral and Prismoidal Arrays

The minimum-satellite, continuous-coverage arrays display a number of important benefits. The first and most obvious is that continuous Earth coverage at any level of redundancy can be achieved using fewer satellites than any other system (including Walker-type arrays). Fewer satellites imply fewer boosters and less ground control and support, compounding the cost savings. The use of elliptic orbits in these prismoidal arrays frees more of the orbital parameters for manipulation than are available for circular orbits, so that a high degree of bias control can be exerted to emphasize performance in selected areas of the world. Continuous coverage is critical for surveillance systems, particularly in the polar regions. Cross-link connectivity is also supported because satellite-to-satellite line of sight is perpetually maintained. Lastly, the satellites of these arrays do not reside in the equatorial regions and so avoid the synchronous equatorial belt, which is presently vastly overcrowded, and their large crossing velocities insure minimum interference intervals in crossing situations with other satellites.

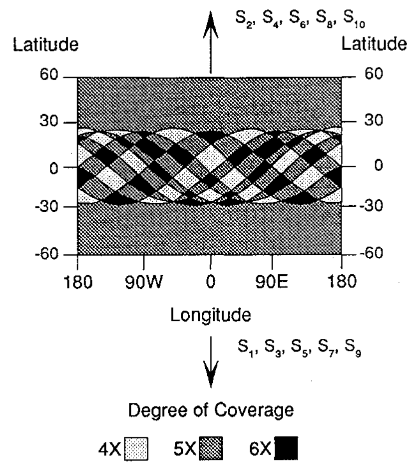

Fig. 17 10-satellite visibility limits, starting position of constellation.

Significance of Results

A new family of satellite arrays has been created that reduces the current minimum-satellite configuration for continuous global coverage by one, resulting in reduced cost for space systems based on such arrays. Higher levels of coverage lead to arrays at very high altitudes, which are unattractive due to the effects of lunar perturbations. However, the single-coverage model and possibly the double-coverage model can be placed at altitudes that are directly applicable to current

Table 5 Satellite periods, eccentricities, apogees, and perigees ($n = 1,...,4$)

$n =$	$T_{c_n} = >$h	$e =$	$a =$ n.mi.	$r_a =$ n.mi.	$r_p =$ n.mi.	$h_a =$ n.mi.	$h_p =$ n.mi.
<1(synch)	24	0.263	22,767	28,755	16,779	25,313	13,337
1	26.49	0.263	24,316	30,711	17,921	27,269	14,479
2	102	0.233	59,735	73,653	45,817	70,211	42,375
3	272	0.218	114,871	139,913	89,829	136,471	86,387
4	568	0.205	187,671	226,144	149,198	222,702	145,756

Table 6 Starting positions, generalized table of orbital parameters for N-tuple continuous-coverage constellations

Satellite number	i, deg	e	ω	Ω	M
1	$f(n)$	$g(n)$	-90 deg	0 deg	0 deg
⋮	↓	↓	⋮	⋮	⋮
k			$(-1)^k(90\text{ deg})$	$-\left(\frac{k-1}{n+1}\right)180$ deg	$+\left(\frac{k-1}{n+1}\right)180$ deg
⋮			⋮	⋮	⋮
$k=n+2$			$(-1)^{n+2}(90\text{ deg})$	-180 deg	$+180$ deg
⋮			⋮	⋮	⋮
$k=2n+2$	↓	↓	$+90$ deg	$-\left(\frac{2n+1}{n+1}\right)180$ deg	$+\left(\frac{2n+1}{n+1}\right)180$ deg

and future missions. Operation of the four-satellite single-coverage array at synchronous periods (23 h, 56 min) instead of 26.49 h degrades the continuous coverage feature very little. At synchronous periods, the amount of continuous coverage is still 99.996%. A majority of the time, complete global coverage is obtained. Also, the four small transient gaps that do occur are localized geographically. By adjusting the orbital parameters, these gaps may be placed in noncritical areas. It is even possible to completely suppress the two gaps occurring in the northern hemisphere, at the expense of slightly enlarging the two corresponding gaps in the southern hemisphere.

Conclusions

The overall conclusions to be drawn from the study on elliptical-orbit constellations are the following:

1) All of the constellations are at altitudes greater than Earth-synchronous. As the degree of redundancy n increases, so does the altitude.

2) From a practical standpoint, only the single- and double-coverage arrays appear useful. The reason for this is that for higher levels of redundancy ($n \geq 3$) the constellation altitudes are so high as to expose the satellite orbits to unacceptable levels of lunar and solar perturbational forces.

Acknowledgment

The four-satellite, common-period, continuous global coverage array and the six-satellite, continuous, double-coverage array were developed independently by STA (U.S. Patent 4854527 and European Patent 0213355 are applicable). The triple and quadruple continuous-coverage models and the generalized theory of prismoidal arrays were developed under the auspices of the Small Business Innovation Research Program, under Contract F04701-88-C-0080. The study sponsor was the U.S. Air Force Space Division, Los Angeles, California. The views expressed in this paper are those of the author and do not necessarily represent U.S. Air Force official position or policy. The assistance of Henry Bowers, in computer programming and data reduction, and Kim Westlake, graphics and layout, for this paper, is gratefully acknowledged.

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